Base-16

Hexadecimal!

Base-16 Hexadecimal

Hexadecimal/Base-16, is convenient for bit patterns

Digits $\{0, 1, 2, \ldots, 9, A, B, \ldots, F\}$

High-order place values come before low-order

- 400_{16} is 4 x 16^2 - 1024
- 020_{16} is 2 x 16^1 - 32
- 006_{16} is 6 x 16^0 - 6

Incrementing a place value beyond 1 causes a **carry**

- 01 + 0F is 10
  - the next higher-order place value increases by one
  - the lower-order place value resets back to zero

Hexadecimal_{16}	Binary_2	Decimal_{10}
0	0000	00
1	0001	01
2	0010	02
3	0011	03
4	0100	04
5	0101	05
6	0110	06
7	0111	07
8	1000	08
9	1001	09
A	1010	10
B	1011	11
C	1100	12
D	1101	13
E	1110	14
F	1111	15

Formalization of *Unsigned* Hex Values

Suppose we define a Base-16 number h, with w place values, (*w stands for width*) as a vector of **hex digits**:

$$\vec{h} = [h_{w-1}, h_{w-2}, \ldots, h_0]$$

We can determine the value of $\vec{d}$ with the following summation:

$$Hex2Unsigned_w\left(\vec{h}\right) = \sum_{i=0}^{w-1} h_i 16^i$$

A concrete example:

$$\vec{h} = [A, F]$$

Hexadecimal$_{16}$	Decimal$_{10}$
0	00
1	01
2	02
3	03
4	04
5	05
6	06
7	07
8	08
9	09
A	10
B	11
C	12
D	13
E	14
F	15

Formalization of *Unsigned* Hex Values

Suppose we define a Base-16 number h, with w place values, (*w stands for width*) as a vector of **hex digits:**

$$\vec{h} = [h_{w-1}, h_{w-2}, \dots, h_0]$$

We can determine the value of $\vec{d}$ with the following summation:

$$Hex2Unsigned_w\left(\vec{h}\right) = \sum_{i=0}^{w-1} h_i 16^i$$

A concrete example:

$$w = 2$$

$$\vec{h} = [A, F] \quad \textit{thus} \quad \begin{matrix} h_1 = A \\ h_0 = F \end{matrix}$$

$$Hex2Unsigned_2([A, F])$$

$$= \sum_{i=0}^{1} h_i 16^i$$

$$= h_0 \times 16^0 + h_1 \times 16^1$$

$$= F \times 16^0 + A \times 16^1$$

$$= 15 \times 1 + 10 \times 16$$

$$= 175$$

$Hexadecimal_{16}$	$Decimal_{10}$
0	00
1	01
2	02
3	03
4	04
5	05
6	06
7	07
8	08
9	09
A	10
B	11
C	12
D	13
E	14
F	15